

HIGGS PARTICLES AT LINEAR e^+e^- COLLIDERS: THEORETICAL ISSUES

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Abstract

I summarize the prospects for discovering and studying the properties of Higgs particles at future high-energy and high-luminosity e^+e^- linear colliders. I will focus on the Higgs particle of the Standard Model and the Higgs bosons predicted by Supersymmetric theories.

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1 Introduction

One of the most important missions of future high-energy colliders will be the search for scalar Higgs particles and the exploration of the electroweak symmetry breaking mechanism. In the Standard Model (SM), one doublet of complex scalar fields is needed to spontaneously break the $SU(2) \times U(1)$ symmetry. Among the four initial degrees of freedom, three Goldstones will be absorbed by the W^\pm and Z bosons to get their masses, and the remaining degree of freedom will correspond to a physical scalar particle, the Higgs boson.¹

Since the couplings of the Higgs boson to fermions and gauge bosons are proportional to the masses of these particles, the only unknown parameter in the SM is the Higgs boson mass, M_H . It is a free parameter and the only things we know about it are that: *i*) it should be larger than ~ 100 GeV from the negative searches at LEP and *ii*) it is probably smaller than ~ 1 TeV, since for higher values the electroweak gauge bosons would interact strongly to insure unitarity in their scattering and perturbation theory would be lost.¹

However, there are both theoretical and experimental hints which indicate that the Higgs boson of the SM might be rather light:

- Global fits of the electroweak precision observables at LEP, SLC and the Tevatron favor a Higgs boson [whose loop contributions to the electroweak parameters depend logarithmically on M_H] with a mass around 100 GeV; an upper bound² of $M_H \lesssim 260$ GeV has been set at the 95% confidence level.

- The quartic Higgs coupling is proportional to M_H^2 and since the scalar sector of the SM is not an asymptotically free theory, the coupling will grow with the energy until it reaches the Landau pole, where the theory does not make sense anymore. If the cut-off Λ where new phenomena should occur is of $\mathcal{O}(1 \text{ TeV})$, the Higgs mass should be smaller than ~ 500 GeV [as verified by simulations on the lattice]. But if one wants to extend the SM up the GUT scale $\Lambda_{\text{GUT}} \sim 10^{16}$ GeV [a prerequisite for the perturbative renormalization of $\sin^2 \theta_W$ from the GUT value $3/8$ down to the experimentally observed value], M_H is restricted to much smaller values. In addition, radiative corrections due to top quark loops could drive the Higgs self-coupling to negative values, therefore destabilizing the vacuum. The stability and the triviality bounds, constrain the SM Higgs boson mass to lie in the range.⁴ $130 \text{ GeV} \lesssim M_H \lesssim 180 \text{ GeV}$.

However, there are two problems that one has to face, when trying to extend the SM to Λ_{GUT} . The first one is the so-called hierarchy or naturalness problem: the Higgs boson tends to acquire a mass of the order of the large scale [the radiative corrections to M_H are quadratically divergent]. The second problem is that the simplest GUTs predict a value for $\sin^2 \theta_W$ that is incompatible with the measured one ~ 0.23 . Low energy Supersymmetry (SUSY)⁵ solves these two problems at once: SUSY particles loops cancel the quadratic divergences to the Higgs boson mass and contribute to the running of the gauge coupling constants to correct for the small discrepancy to the observed value of $\sin^2 \theta_W$.

The minimal supersymmetric extension of the Standard Model (MSSM) requires the existence of two isodoublets of Higgs fields,¹ leading to three neutral, h/H (CP-even with h being the lightest particle), A (CP-odd) and a pair of charged scalar particles H^\pm . Besides the four masses, two additional parameters define the properties of these particles: a mixing angle α in the neutral CP-even sector and $\tan \beta$ the ratio of the two vacuum expectation values of the Higgs fields, which from GUT restrictions is assumed in the range $1 < \tan \beta < m_t/m_b$. Supersymmetry leads to several relations among these parameters and only two of them [taken in general to be $\tan \beta$ and M_A] are in fact independent. These relations impose a strong hierarchical structure of the mass spectrum and lead to the definite prediction that, at the tree level, the neutral h boson should be lighter than the Z boson.

However, radiative corrections involving mainly the top quark and its SUSY partners, introduce new [soft SUSY-breaking] parameters in the Higgs sector and affect the Higgs boson masses and couplings significantly. The leading part of these corrections grows as the fourth power of the top quark mass and logarithmically with the common squark mass [a strong dependence on the trilinear stop coupling A_t is also present], and shift the mass of the lightest h boson upwards. A recent calculation,⁶ performed at the two-loop level in the diagrammatic approach, restrict the h boson mass to be less than ~ 135 GeV.

Note that for large values of M_A , the heavy neutral and charged Higgs bosons are nearly mass degenerate, while the lightest h boson reaches its maximal mass value. One is then in the so-called decoupling regime where the lightest h boson has almost the same properties as the standard Higgs boson [but with a mass below ~ 135 GeV] and the SM and MSSM Higgs sectors look practically the same.

In more general SUSY scenarii, one can add an arbitrary number of Higgs doublet and/or singlet fields without being in conflict with high precision data.¹ The Higgs spectrum becomes then much more complicated than in the MSSM, and much less constrained. However, the triviality argument always imposes a bound on the mass of the lightest Higgs boson of the theory. For instance, if only one Higgs singlet field is added to the MSSM, an upper bound $M_h \lesssim 150$ GeV can be derived.⁷ In the most general SUSY model, with arbitrary matter content and gauge coupling unification near the GUT scale, and absolute upper limit on the mass of the lightest Higgs boson, $M_h \lesssim 205$ GeV, has been recently derived.⁸

Thus, either in the SM or in its SUSY extensions, a Higgs boson should be lighter than ~ 200 GeV, and will be therefore kinematically accessible at an e^+e^- linear collider with a c.m. energy $\sqrt{s} \gtrsim 350$ GeV. In this talk, I will summarize the prospects for such a collider to discover and to study the properties of this particle.⁹

2 Higgs Boson Decays and Production

In the SM, the profile of the Higgs particle [decay widths, branching ratios and production cross sections] is uniquely determined once M_H is fixed. The profile of the MSSM Higgs bosons is determined to a large extent also by their couplings to fermions and gauge bosons, which in general depend strongly on $\tan\beta$ [and α].

2.1 Higgs decays

In the “low mass” range $M_H \lesssim 130$ GeV, the SM Higgs boson decays¹⁰ into a large variety of channels, the main mode being by far into $b\bar{b}$ pairs with a BR of $\sim 90\%$ followed by the decays into $c\bar{c}$ and $\tau^+\tau^-$ pairs with BRs of $\sim 5\%$. Also of significance is the top-loop mediated Higgs decay into gluons which for $M_H \sim 120$ GeV occurs at the level of $\sim 5\%$. The top and W -loop mediated $\gamma\gamma$ and $Z\gamma$ decay modes are very rare, the BRs being of $\mathcal{O}(10^{-3})$; however the $\gamma\gamma$ decays lead to clear signals and are interesting being sensitive to new heavy particles. Note that QCD corrections to the hadronic decays turn out to be quite substantial, and together with the rather imprecise present knowledge of the strong coupling constant α_s and the c and b quark masses, introduce some uncertainties in the BRs.

In the “high mass” range $M_H \gtrsim 130$ GeV, the H bosons decay into WW and ZZ pairs, with one of the gauge bosons being virtual below the threshold. Above the ZZ threshold, the Higgs boson decays almost exclusively into these channels with BRs of $2/3$ for WW and $1/3$ for ZZ [for high M_H values, the opening of the $t\bar{t}$ channel does not alter significantly this pattern].

In the low mass range, the H boson is very narrow $\Gamma_H \lesssim 10$ MeV, but the width becomes rapidly wider for masses larger than 130 GeV, reaching 1 GeV at the ZZ threshold. The Higgs decay width cannot be measured directly for $M_H \lesssim 200$ GeV.

The decay pattern of the MSSM Higgs bosons depends strongly on $\tan\beta$. For large $\tan\beta$ values, it is simple a result of the strong enhancement of the Higgs couplings to down-type fermions: the neutral Higgs bosons will decay into $b\bar{b}$ ($\sim 90\%$) and $\tau^+\tau^-$ ($\sim 10\%$) pairs, and H^\pm into $\tau\nu_\tau$ pairs below and $t\bar{b}$ pairs above the top-bottom threshold. Only when M_h approaches its maximal value is this simple rule modified since in this decoupling limit, the h boson decays as the SM Higgs boson. For small values of $\tan\beta$, the decay pattern of the heavy neutral Higgs bosons can be more complicated. The b decays are in general not dominant any more; instead, cascade decays to pairs of light Higgs bosons and mixed pairs of Higgs and gauge bosons are important and decays to WW/ZZ pairs will play a role. For very large masses, they decay almost exclusively to top quark pairs. The decay pattern of the charged Higgs bosons for small $\tan\beta$ is similar to that at large $\tan\beta$ except in the intermediate mass range where cascade decays to Wh are dominant. In addition, below threshold three-body decays might be important.

When the decays into SUSY particles are kinematically allowed [for the heavy Higgs scalars] the pattern becomes even more complicated since the decay channels into charginos, neutralinos and squarks might be non-negligible.

In more general SUSY scenarii, the decays of the Higgs bosons can be much more complicated than in the MSSM; however, this does not lead to any difficulty to detect some of the particles at e^+e^- colliders as will be discussed later.

2.2 Higgs production

The main production mechanism⁹ of the SM Higgs particles in e^+e^- collisions are the Higgs-strahlung process, $e^+e^- \rightarrow (Z^*) \rightarrow ZH$ [with a cross section which scales as $1/s$ and therefore dominates at low energies], and the WW fusion mechanism, $e^+e^- \rightarrow \nu\bar{\nu}(W^*W^*) \rightarrow \nu\bar{\nu}H$ [with a cross section rising like $\log(s/M_H^2)$ and which dominates at high energies]. The cross section for the ZZ fusion mechanism, $e^+e^- \rightarrow e^+e^-H$, is an order of magnitude smaller than the later due to the smallness of the NC couplings compared to the CC ones, but gives some complementary information.¹¹ There are also higher order processes: associated Higgs production with top quarks or a photon and double Higgs production in the strahlung and fusion processes or through loops; they have smaller cross sections but are very useful when it comes to study the Higgs properties as will be discussed later. Additional production mechanisms are also provided by the $\gamma\gamma \rightarrow H$ and $e\gamma \rightarrow \nu WH$ processes, the high-energy photons generated by Compton back scattering of laser light; they are discussed elsewhere.¹²

At $\sqrt{s} \sim 500$ GeV, the Higgs-strahlung and the WW fusion processes have approximately the same cross sections for the mass range $100 \text{ GeV} \lesssim M_H \lesssim 200$ GeV. With a luminosity $\int \mathcal{L} \sim 500 \text{ fb}^{-1}$ as it is expected for the TESLA design,¹³ a sample of ~ 75.000 Higgs bosons can be collected in a one year running for $M_H \sim 130$ GeV. Assuming that 25 events are required to establish a discovery [the signal with $H \rightarrow b\bar{b}$ is easy to detect at e^+e^- colliders, especially with efficient micro-vertex detectors¹⁴] less than one hour running is needed in such a machine [to be compared with the much longer running time at the LHC for the $H \rightarrow \gamma\gamma$ mode¹⁵]. Thus, the discovery of the SM Higgs particle [if kinematically allowed] is not a problem in the clean environment of such an e^+e^- collider.

In the MSSM, besides the usual Higgs-strahlung and fusion processes for the production of the CP-even Higgs bosons h and H , the neutral Higgs particles can also be produced pairwise: $e^+e^- \rightarrow A + h/H$. The cross sections for the Higgs-strahlung and the pair production as well as the cross sections for the production of h and H are mutually complementary, coming either with a coefficient $\sin^2(\beta - \alpha)$ or $\cos^2(\beta - \alpha)$. The sum of the cross sections for h production in the strahlung and associated processes is roughly the same as the cross section for the SM Higgs boson [with the same mass] in the strahlung process. The CP-even Higgs particles can also be searched for in the WW and ZZ fusion mechanisms. Charged Higgs bosons can be produced pairwise, $e^+e^- \rightarrow H^+H^-$, through γ, Z exchange and the cross section which depends only on M_{H^\pm} , is large up to $M_{H^\pm} \sim 230$ GeV; the H^\pm bosons can also be produced in top decays if kinematically allowed.

The discussion on the MSSM Higgs production at e^+e^- linear colliders can be summarized in the following points⁹: *i*) The Higgs boson h can be detected in the entire range of the MSSM parameter space, either through the bremsstrahlung process or through pair production; in fact, this conclusion holds true even at a c.m. energy of 300 GeV and with a luminosity of a few fb^{-1} . *ii*) All SUSY Higgs bosons can be discovered at a 500 GeV collider if the H, A and H^\pm masses are less than ~ 230 GeV; for higher masses, one simply has to increase the c.m. energy. *iii*) Even if the decay modes of the Higgs bosons are very complicated [e.g. they decay invisibly] missing mass techniques allow their detection.

In extensions of the MSSM, the Higgs production processes are as the ones above but the phenomenological analyses are more involved since there is more freedom in the choice of parameters. However, even if the Higgs sector is extremely complicated, there is always a light Higgs boson which has sizeable couplings to the Z boson. This Higgs particle can be thus produced in the strahlung process, $e^+e^- \rightarrow Z + "h"$, and using the missing mass technique this " h " particle can be detected. Recently a "no-loose theorem" has been proposed!¹⁶ a Higgs boson in SUSY theories can be always detected at a 500 GeV e^+e^- collider with a luminosity of $\int \mathcal{L} \sim 500 \text{ fb}^{-1}$ in the strahlung process, regardless of the complexity of the Higgs sector of the theory and of the decays of the Higgs boson.

3 Precision Measurements at the LC

Thus a light Higgs boson can be found without any problem at a future linear collider. However, such a particle might be first discovered at the present machines LEP¹⁸ and the Tevatron,¹⁹ or at the LHC.¹⁵ As discussed by M. Peskin in the introductory talk,¹⁷ the job of a linear e^+e^- collider, will be rather, to study the properties of the Higgs particles. The clean environment and the very high luminosities which are expected [e.g. $\int \mathcal{L} \gtrsim 100 \text{ fb}^{-1}$ for the TESLA design], allow to study these properties in great details and to make very accurate measurements in the Higgs sector. We summarize below the measurements which can be made in the main production mechanisms as well as in the higher-order processes. We will focus on the case of the SM Higgs boson, which is equivalent for $M_H \lesssim 130 \text{ GeV}$, to the case of the light h boson of the MSSM close to the decoupling regime. A more quantitative discussion will be given by S. Yamashita.¹⁴

3.1 Measurements in the main processes

- The measurement of the recoil e^+e^- or $\mu^+\mu^-$ mass in the Higgs-strahlung process, $e^+e^- \rightarrow ZH \rightarrow He^+e^-$ and $H\mu^+\mu^-$, allows a very good determination of the Higgs boson mass. At $\sqrt{s} = 350 \text{ GeV}$ and with a luminosity of $\int \mathcal{L} = 500 \text{ fb}^{-1}$, a precision of $\sim 150 \text{ MeV}$ can be reached²⁰ for a Higgs boson mass of $M_H \sim 120 \text{ GeV}$. The precision can be significantly increased if one uses the hadronic decays of the Z boson [which have more statistics] but since the mass resolution is rather bad in the simplest way, one has to make some kinematical fits of 4-jets with distributions.²¹ A threshold scan might also improve the measurement. The one per mile accuracy which can be obtained for the Higgs boson mass can be very important, especially in the MSSM where it allow to strongly constrain the other parameters of the model.

- The angular distribution of the Z/H in the Higgs-strahlung process is sensitive to the spin-zero of the Higgs particle: at high-energies the Z is longitudinally polarized and the distribution follows the $\sim \sin^2\theta$ law which unambiguously characterizes the production of a $J^P = 0^+$ particle. The spin-parity quantum numbers of the Higgs bosons can also be checked experimentally by looking at correlations in the production $e^+e^- \rightarrow HZ \rightarrow 4\text{-fermions}$ or decay $H \rightarrow WW^* \rightarrow 4\text{-fermions}$ processes, as well as in the more difficult channel $H \rightarrow \tau^+\tau^-$ for $M_H \lesssim 140 \text{ GeV}$. An unambiguous test of the CP nature of the Higgs bosons can be made in the process $e^+e^- \rightarrow t\bar{t}H$ or at laser photon colliders in the loop induced process $\gamma\gamma \rightarrow H$.

- The masses of the gauge bosons are generated through the Higgs mechanism and the Higgs couplings to these particles are proportional to their masses. This fundamental prediction has to be verified experimentally. The Higgs couplings to ZZ/WW bosons can be directly determined by measuring the production cross sections in the bremsstrahlung and the fusion processes. In the $e^+e^- \rightarrow He^+e^-$ and $H\mu^+\mu^-$ processes, the total cross section can be measured²⁰ with a precision less than $\sim 3\%$ at $\sqrt{s} = 350$ GeV and with $\int \mathcal{L} = 500 \text{ fb}^{-1}$. This leads to an accuracy of $\sim 1.5\%$ on the HZZ coupling.

- The measurement of the branching ratios of the Higgs boson are of utmost importance. For Higgs masses below $M_H \lesssim 130$ GeV a large variety of BRs can be measured at the linear collider. The $b\bar{b}, c\bar{c}$ and $\tau^+\tau^-$ BRs allow to measure the relative couplings of the Higgs bosons to these fermions and to check the fundamental prediction of the Higgs mechanism that they are proportional to fermion masses. In particular $\text{BR}(H \rightarrow \tau^+\tau^-) \sim m_\tau^2/3\bar{m}_b^2$ allows to make such a test. In addition, these branching ratios, if measured with enough accuracy, could allow to distinguish a Higgs boson in the SM from its possible extensions. The gluonic BR is sensitive to the $t\bar{t}H$ Yukawa coupling [and might therefore give an indirect measurement of this important coupling] and to new strongly interacting particles which couple to the Higgs boson [such as top squarks in SUSY extensions of the SM]. The branching ratio into W boson starts to be significant for Higgs masses of the order of 120 GeV and allows to measure the HWW coupling. The BR of the loop induced $\gamma\gamma$ decay of the Higgs boson is also very important since it is sensitive to new particles [the measurement of this BR gives the same information as the measurement of the cross section for Higgs boson production at $\gamma\gamma$ colliders].

In this workshop, a lot of experimental work has been performed to assess the level of precision with which all these BRs can be measured,¹⁴ and the results are very impressive. Table 1 from Ref.²² summarizes the achieved precision at $\sqrt{s} = 350$ GeV and with $\int \mathcal{L} = 500 \text{ fb}^{-1}$ [details can be found in¹⁴]. These errors are so small that one can tell a SM Higgs boson from the MSSM h boson [whose couplings to fermions and gauge bosons are in principle altered by mixing angle factors] up to a pseudoscalar Higgs boson mass of $M_A \sim 700$ GeV. In fact, the experimental errors are even smaller than the theoretical errors which affect some BRs [in particular for the gg and $c\bar{c}$ modes²²] due to the [mostly experimental...] uncertainties in the measurement of α_s, m_c and to a lesser extent m_b .

$b\bar{b}$	$c\bar{c}$	gg	$\tau^+\tau^-$	W^+W^-	$\gamma\gamma$
2%	8%	6%	6%	2-10%	20%

Table 1: Expected accuracies on Higgs BR's at $\sqrt{s}350$ GeV and with $\int \mathcal{L} = 500 \text{ fb}^{-1}$.

- As discussed previously, the total width of the Higgs boson [for masses less than ~ 200 GeV] is so small that it cannot be resolved experimentally. However, the measurement of $\text{BR}(H \rightarrow WW)$ allows an indirect determination of Γ_H since the HWW coupling can be determined from the measurement of the Higgs production cross section in the WW fusion process [or from the measurement of the cross section of the Higgs-strahlung process, assuming SU(2) invariance]. The accuracy of the Γ_H measurement follows then from that of the WW branching ratio.²²

3.2 Measurements in higher order processes

There are several processes where Higgs particles are produced in pairs or in association with heavy particles or else through loop diagrams. Since these processes are of higher order in perturbation theory, the production rates are in general rather small, at or below the femtobarn level. Very high luminosities $\int \mathcal{L} \sim 1 \text{ ab}^{-1}$ offer a unique opportunity to study these processes and to gain additional information on the Higgs sector. Some of these processes have been discussed in the parallel sessions and the main points are summarized below:

- Associated production of Higgs bosons with top quark pairs $e^+e^- \rightarrow t\bar{t}H$:

The Higgs coupling to top quarks, which is the largest coupling in the electroweak SM, is directly accessible in this process.²³ In addition to Higgs radiation from the quark lines which gives access to the $t\bar{t}H$ Yukawa coupling, there is also Higgs emission from the Z line [and diagrams with the exchange of heavier Higgs bosons in the MSSM], which nevertheless give small contributions to the production cross section. The later is at the femtobarn level for $M_H \sim 100 \text{ GeV}$ at a c.m. energy of 500 GeV, but the signal is quite spectacular [two W bosons and four b quarks, with kinematical constraints to reconstruct the top quarks and the H boson] giving the possibility of isolating these events with a luminosity of $\mathcal{O}(1 \text{ ab}^{-1})$. A recent analysis,²⁴ with detailed simulations of the signal and backgrounds including realistic detector effects and reconstruction procedures, has shown that an accuracy of $\sim 5\%$ can be achieved in the measurement of the $t\bar{t}H$ coupling for a mass $M_H \simeq 120 \text{ GeV}$ at a c.m. energy $\sqrt{s} = 800 \text{ GeV}$ and with a luminosity $\int \mathcal{L} = 1 \text{ ab}^{-1}$. The QCD corrections to this process have been calculated recently.²⁵ They are large and positive [with K -factors of the order of 1.4 to 2.4] at $\sqrt{s} \sim 500 \text{ GeV}$ because of resonance effects, and small and negative [with K -factors of order 0.8–0.9] at $\sqrt{s} \sim 1 \text{ TeV}$. Note that the associated production of Higgs bosons with $b\bar{b}$ pairs can have a significant cross section in the MSSM, for large $\tan\beta$ values and a low pseudoscalar A mass; in this case this processes would allow a nice direct determination²⁶ of the important parameter $\tan\beta$ [which is difficult to achieve in other processes].

- Double Higgs production in the strahlung process $e^+e^- \rightarrow HHZ$:

To establish the Higgs mechanism experimentally in an unambiguous way, the self-energy potential of the Higgs field must be reconstructed. This requires the determination of the trilinear [and quadrilinear] self-couplings as predicted for instance in the SM or MSSM. These couplings can be probed in the production of pairs of neutral Higgs bosons. A coherent picture of the trilinear couplings has been given here²⁷ with the production of pairs of neutral Higgs bosons in the SM and MSSM, in all relevant channels of double Higgs-strahlung, associated multi-Higgs production and WW/ZZ fusion to Higgs boson pairs. The most interesting process at energies around 500 GeV is the double Higgs-strahlung process, $e^+e^- \rightarrow HHZ$. The cross section, which is very sensitive to the trilinear self-coupling, is of the order 0.5 fb for $M_H \sim 100 \text{ GeV}$. This leads to approximately one thousand events for a luminosity of $\int \mathcal{L} = 2 \text{ ab}^{-1}$ [corresponding to 4 years of running with the expected luminosity at TESLA] with an extremely clean signal [a Z boson with 4 b -jets with two $b\bar{b}$ pairs having an invariant mass M_H which is expected to be measured precisely in the main production processes]. A detailed simulation²⁸ has shown that the trilinear coupling can be measured with a precision at the $\sim 15\%$ level. This

analysis is preliminary and not yet optimized, and a better determination is to be expected in the future. At higher energies, double Higgs production in the WW fusion channel, $e^+e^- \rightarrow \nu\bar{\nu}HH$, which has a larger cross section [~ 1 fb at $\sqrt{s} = 1.5$ TeV for $M_H \sim 100$ GeV], might be used. The quadrilinear Higgs self-coupling can be measured in triple Higgs boson production, but the cross section is suppressed by an additional electroweak factor, and is therefore too small to be observable.²⁹

- Associated production of a Higgs boson and a photon, $e^+e^- \rightarrow H\gamma$:

In the SM, this process proceeds through s -channel $\gamma^*\gamma H$ and $Z^*\gamma H$ vertex diagrams, but additional t -channel vertex and box diagrams involving W /neutrino and Z /electron exchange also occur. It is therefore sensitive to the $H\gamma\gamma$ and $HZ\gamma$ vertices. These couplings do not occur at the tree level but are induced by loops of heavy particles, which if their interaction with the Higgs boson is proportional to their masses, do not decouple for very large masses. These vertices could therefore serve to count the number of particles which couple to the H boson. [These couplings can be also accessed in the decays $H \rightarrow \gamma\gamma$ and $H \rightarrow Z\gamma$ but the BRs are very small $\sim 10^{-3}$; the $H\gamma\gamma$ coupling can also be determined directly by means of the laser $\gamma\gamma \rightarrow H$ fusion process]. A precise determination of these couplings could help to distinguish between the SM Higgs boson and Higgs particles predicted by some of its extensions such a two-Higgs Doublet Model³⁰ [in these models, it is also useful for h and A production in the process $Z \rightarrow \gamma$ + Higgs with the linear collider running on the Z -resonance,³⁰ since the experimental bounds on the masses are not as tight as in the MSSM] or supersymmetric theories³¹ [where scalar tops and charginos loops might have some significant contributions]. Unfortunately, the cross section is rather small: at a 500 GeV collider it is of $\mathcal{O}(0.1$ fb), leading to two hundred events with $\int \mathcal{L} = 2$ ab⁻¹. This number would allow, roughly, a measurement of the cross section at the 10% level. The monochromatic photon makes the signal very clean, but not detailed simulation has been performed yet to access the viability of this signal.

- Associated production of Higgs bosons with top squark pairs, $e^+e^- \rightarrow \tilde{t}\tilde{t}h$:

In the MSSM, if the mixing between third generation squarks is large, scalar top [and also bottom] quarks can be rather light and at the same time, their coupling to Higgs bosons can become substantial. For instance, the couplings of the lightest stops to the h boson are proportional to $g_{h\tilde{t}_1\tilde{t}_1} \sim A_t - \mu/\tan\beta$, and large values of this couplings might have a rather strong impact on the phenomenology of the MSSM Higgs bosons.³² The measurement of this important coupling would open a window to probe directly some of the soft-SUSY breaking terms of the potential. To measure Higgs-stop couplings directly, one needs to consider the three-body associated production of Higgs bosons with stop pairs [the SUSY analog to the $t\bar{t}h$ associated production process]. At future linear e^+e^- colliders, the final state $\tilde{t}_1\tilde{t}_1h$ may be generated in three ways:³³ (i) two-body production of a mixed pair of top squarks and the decay of the heaviest stop to the lightest one and a Higgs boson, (ii) the continuum production in e^+e^- annihilation $e^+e^- \rightarrow \tilde{t}_1\tilde{t}_1h$ and (iii) the continuum production in $\gamma\gamma$ collisions $\gamma\gamma \rightarrow \tilde{t}_1\tilde{t}_1h$. In the continuum production in e^+e^- collisions at $\sqrt{s} \sim 800$ GeV, the cross sections can exceed 1 fb for not too large \tilde{t}_1 masses [$\lesssim 200$ GeV] and large values of the parameter A_t [$\gtrsim 1$ TeV] and is thus comparable to the one of SM-like process $e^+e^- \rightarrow t\bar{t}h$. This provides

more than one thousand events in a few years, with a luminosity $\int \mathcal{L} \sim 500 \text{ fb}^{-1}$, which should be sufficient to isolate the final state and measure $g_{\tilde{t}_1 \tilde{t}_1 h}$ with some accuracy. Note that in most part of the MSSM parameter space, the final state topology will consist of $4b$ quarks, two of them peaking at an invariant mass M_h , two real or virtual W bosons and missing energy [i.e. the same topology as the process $e^+e^- \rightarrow t\bar{t}h$, except for the missing energy].

4 Conclusions

In the Standard Model, global fits of the electroweak data favor a light Higgs boson, $M_H \lesssim 260 \text{ GeV}$, and if the theory is to remain valid up to the GUT scale, the Higgs boson should be lighter than 200 GeV . In supersymmetric extensions of the SM, there is always one light Higgs boson with a mass $M_h \lesssim 135 \text{ GeV}$ in the minimal version and $M_h \lesssim 205 \text{ GeV}$ in the most general one. Thus, a Higgs particle is definitely accessible at a linear e^+e^- collider with a c.m. energy of $\sqrt{s} \gtrsim 350 \text{ GeV}$.

The detection of such a particle is not a problem at e^+e^- colliders. The search can be made in a large variety of channels: Higgs-strahlung and vector boson fusion processes in the SM, while additional processes are provided by Higgs pair production in SUSY extensions. The cross sections give large samples of events, especially if a very high luminosity, $\int \mathcal{L} \gtrsim 100 \text{ fb}^{-1}$, is available. The signals are very clear in the clean environment of e^+e^- colliders, and the possibility of making efficient b -tagging, using missing mass techniques and the polarization of the initial beams, makes the search even easier.

The very high luminosities expected in some machines and the very clean environment allow to investigate thoroughly the properties of the discovered Higgs bosons. In the main production processes, the Higgs boson mass and width, the spin and parity quantum numbers and the couplings to gauge bosons and fermion can be measured. Higher order processes allow the direct determination of some very important couplings such as the Higgs- $t\bar{t}$ Yukawa coupling, the trilinear Higgs self-coupling, the couplings to photons and possibly, in supersymmetric extensions of the SM, the coupling to top squarks.

In conclusion: a future linear e^+e^- collider with a c.m. energy $\sqrt{s} \gtrsim 350 \text{ GeV}$ and a luminosity $\int \mathcal{L} \gtrsim 100 \text{ fb}^{-1}$ is an ideal instrument to search for Higgs bosons and to explore thoroughly the electroweak breaking mechanism.

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References

1. For a review of the Higgs sector in the SM and MSSM and references on original work; J.F. Gunion, H.E. Haber, G.L. Kane, S. Dawson, *The Higgs Hunter's Guide*, Addison-Wesley, Reading 1990.
2. ALEPH Collaboration, hep-ex/9908016; DELPHI Collaboration, CERN-EP-99-006; L3 Collaboration, hep-ex/9909004; OPAL Collaboration, hep-ex/9908002; LEPC meeting, september 1999.
3. The LEP Collaborations, CERN-EP-99-015.

4. T. Hambye and K. Riesselmann, Phys. Rev. D55 (1997) 7255; G. Altarelli and G. Isidori, Phys. Lett. B337 (1994) 341; J. Casas, J. Espinosa and M. Quiros, Phys. Lett. B382 (1996) 374; M. Sher, Phys. Lett. B317 (1993) 159.
5. See e.g., H. Haber and G. Kane, Phys. Rep. 117 (1985) 75.
6. See the talk of G. Weiglein, hep-ph/9910283, these proceedings.
7. M. Drees, Int. J. Mod. Phys. A4 (1989) 87; U. Ellwanger, M. Rausch de Traubenberg and C. A. Savoy, Nucl. Phys. B492 (1997) 21.
8. J.R. Espinosa and M. Quiros, Phys. Rev. Lett. 81 (1998) 516.
9. For earlier discussions of Higgs physics at e^+e^- colliders and for a complete set of references, see: E. Accomando et al., Phys. Rep. 299 (1998) 1; P.M. Zerwas et al., ECFA-DESY Workshop, hep-ph/9605437; NLC and NLC ZDR Design and Physics Working Groups, hep-ex/9605011; A. Djouadi, hep-ph/9605426 and hep-ph/9512312; H.E. Haber, Proc. "Physics and Experiments with Linear Colliders" Saariselkä, Finland, 1991; F. Zwirner, *ibid*; S. Komamiya *ibid*; J.F. Gunion, "Physics and Experiments with Linear Colliders", Waikoloa, Hawaii, 1993; P. Janot, *ibid*; Y. Okada, "Physics and Experiments with Linear Colliders", Morioka, Japan, 1993; R. van Kooten, *ibid*.
10. For recent studies, see: A. Djouadi, J. Kalinowski and M. Spira, Comp. Phys. Commun. 108 (1998) 56; A. Djouadi, M. Spira and P. Zerwas, Z. Phys. C70 (1996) 427; A. Djouadi, J. Kalinowski and P. Zerwas, Z. Phys. C70 (1996) 435; A. Djouadi, hep-ph/9903382; M. Spira, hep-ph/9705337.
11. See P. Minkowski, these proceedings.
12. See the $\gamma\gamma$ session, these proceedings.
13. See e.g. the talk of R. Brinkman, these proceedings.
14. For a summary of experimental results: see S. Yamashita, these proceedings.
15. ATLAS Collaboration, Technical Proposal, Report CERN-LHCC 94-43; CMS Collaboration, Technical Proposal, Report CERN-LHCC 94-38.
16. J. R. Espinosa and J. F. Gunion, Phys. Rev. Lett. 82 (1999) 1084.
17. M. Peskin, these proceedings.
18. M. Carena and P.M. Zerwas et al, in "Physics at LEP2", hep-ph/9602250.
19. See the talk of J. Conway, these proceedings.
20. W. Lohmann and P. Garcia-Abia, these proceedings.
21. A. Juste, these proceedings.
22. M. Battaglia; G. Borisov and F. Richard, these proceedings.
23. A. Djouadi, J. Kalinowski and P. Zerwas, Z. Phys. C54 (1992) 255.
24. A. Juste and G. Merino, hep-ph/9910301, these proceedings.
25. S. Dawson and L. Reina, Phys. Rev. D59 (1999) 4012; *ibid* D60 (1990) 5003 and these proceedings; S. Dittmaier et al., Phys. Lett. B441 (1998) 383.
26. A. Sopczak, these proceedings.
27. W. Kilian, these proceedings.
28. P. Bambade, P. Gay and P. Lutz, ECFA-DESY Workshop.
29. W. Kilian et al., Eur. Phys. J. C10 (1999) 27.
30. M. Krawczyk, these proceedings.
31. A. Djouadi, V. Driesen, W. Hollik, J.I. Illana, Eur. Phys. J. C1 (1998) 149.
32. For reviews: A. Djouadi, hep-ph/9901237; G. Bélanger et al., hep-ph/9907207.
33. J.L. Kneur et al., hep-ph/9910269, these proceedings.